

Supplement to: A Bayesian approach for some zero-modified Poisson mixture models

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Abstract: Supplementary material for the paper “A Bayesian approach for some
zero-modified Poisson mixture models”.

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1 Simulation studies

This section contains a full report regarding the obtained results in each simulation study proposed in the paper. The goals and particularities of each study are presented in Section 6 of the manuscript. The abbreviations used in the figures and tables are listed in the following in alphabetical order.

- ANCP: above noncoverage probability;
- BCI: Bayesian credible interval;
- BNCP: below noncoverage probability;
- BRE: Bayesian relative efficiency;
- CP: coverage probability;
- MAPE: mean absolute percentage error;
- MSE: mean squared error;
- VAR: variance;
- ZMPL: zero-modified Poisson-Lindley;
- ZMPS_h: zero-modified Poisson-Shanker;
- ZMPS_u: zero-modified Poisson-Sujatha.

1.1 Study 1

1.1.1 Zero-inflated artificial data

The following tables illustrate the results obtained in the first simulation study, considering scenarios 1-4 of the zero-inflated case as described in Section 6 (Table 7) of the paper.

Table 1: Summary of the Bayesian estimation procedure for zero-inflated samples (ZMPL model), $\theta = 0.50$.

n	Parameter	BIAS	VAR	MSE	MAPE (%)
Scenario 1					
50	θ	0.1080	0.0606	0.0723	36.2519
	ω	0.0034	0.0029	0.0029	26.2943
	p	0.0171	0.0048	0.0051	28.1300
100	θ	0.0455	0.0205	0.0226	20.6406
	ω	0.0021	0.0014	0.0014	18.7453
	p	0.0083	0.0024	0.0024	19.7549
200	θ	0.0252	0.0095	0.0102	14.1610
	ω	0.0006	0.0007	0.0007	12.7316
	p	0.0039	0.0011	0.0011	13.5143
500	θ	0.0094	0.0029	0.0030	8.5523
	ω	-0.0006	0.0003	0.0003	7.8953
	p	0.0005	0.0004	0.0004	8.3193
Scenario 2					
50	θ	0.0274	0.0088	0.0095	14.8179
	ω	-0.0033	0.0048	0.0048	8.4442
	p	0.0096	0.0088	0.0089	9.1931
100	θ	0.0133	0.0042	0.0044	10.0023
	ω	0.0003	0.0021	0.0021	5.5269
	p	0.0071	0.0039	0.0040	6.2365
200	θ	0.0074	0.0018	0.0019	6.6842
	ω	0.0003	0.0010	0.0010	3.9307
	p	0.0041	0.0020	0.0020	4.5188
500	θ	0.0037	0.0006	0.0006	3.9766
	ω	0.0001	0.0005	0.0005	2.5635
	p	0.0020	0.0008	0.0008	2.7969

Table 2: Coverage probabilities of the BCIs using zero-inflated samples (ZMPL model), $\theta = 0.50$.

n	Parameter	BNCP	CP	ANCP	BNCP	CP	ANCP
		Scenario 1			Scenario 2		
50	θ	4.00	93.40	2.60	3.60	93.00	3.40
	ω	2.00	94.20	3.80	3.40	94.00	2.60
	p	2.40	95.40	2.20	5.00	92.80	2.20
100	θ	1.80	96.00	2.20	3.20	94.40	2.40
	ω	2.60	94.00	3.40	3.20	94.80	2.00
	p	2.40	94.80	2.80	3.00	95.80	1.20
200	θ	3.00	94.40	2.60	3.40	94.60	2.00
	ω	1.20	96.60	2.20	2.20	95.40	2.40
	p	2.00	96.40	1.60	2.00	96.00	2.00
500	θ	2.20	95.40	2.40	1.20	96.20	2.60
	ω	2.00	95.20	2.80	2.60	95.20	2.20
	p	2.80	94.20	3.00	3.00	94.80	2.20

Table 3: Summary of the Bayesian estimation procedure for zero-inflated samples (ZMPL model), $\theta = 5.00$.

n	Parameter	BIAS	VAR	MSE	MAPE (%)
Scenario 3					
50	θ	1.0677	14.8947	16.0346	58.5396
	ω	0.0166	0.0006	0.0009	57.7185
	p	0.1926	0.1711	0.2082	127.1419
100	θ	2.6378	31.2366	38.1944	86.2703
	ω	0.0057	0.0003	0.0004	39.9392
	p	0.1759	0.1412	0.1721	121.5846
200	θ	4.4031	67.4253	86.8126	114.9686
	ω	0.0017	0.0002	0.0002	27.6028
	p	0.1908	0.1510	0.1874	122.8802
500	θ	2.7461	47.9365	55.4774	74.9845
	ω	0.0004	0.0001	0.0001	17.2483
	p	0.1023	0.0600	0.0705	71.3279
Scenario 4					
50	θ	4.0657	57.0500	73.5797	107.4576
	ω	0.0032	0.0028	0.0028	27.7432
	p	0.6478	1.8494	2.2691	108.3521
100	θ	3.6657	82.2925	95.7300	93.6567
	ω	0.0017	0.0014	0.0014	19.7314
	p	0.5323	1.7420	2.0254	87.0143
200	θ	1.8732	43.0111	46.5200	53.6183
	ω	0.0002	0.0006	0.0006	13.2571
	p	0.2692	0.7824	0.8549	50.5409
500	θ	0.5331	2.4133	2.6975	22.9489
	ω	-0.0007	0.0002	0.0002	8.1805
	p	0.0756	0.0573	0.0630	22.5011

Table 4: Coverage probabilities of the BCIs using zero-inflated samples (ZMPL model), $\theta = 5.00$.

n	Parameter	BNCP	CP	ANCP	BNCP	CP	ANCP
		Scenario 3			Scenario 4		
50	θ	0.00	91.20	8.80	1.60	93.40	5.00
	ω	5.20	94.80	0.00	3.20	92.00	4.80
	p	3.20	96.00	0.80	3.00	91.20	5.80
100	θ	0.00	93.20	6.80	5.60	91.00	3.40
	ω	3.40	96.60	0.00	2.40	94.00	3.60
	p	2.60	94.60	2.80	5.00	92.00	3.00
200	θ	2.40	92.20	5.40	3.20	94.20	2.60
	ω	0.80	96.20	3.00	1.20	96.80	2.00
	p	3.20	92.00	4.80	3.80	93.60	2.60
500	θ	3.00	93.00	4.00	2.80	95.20	2.00
	ω	2.00	95.60	2.40	1.40	96.40	2.20
	p	4.00	91.00	5.00	3.20	94.40	2.40

Table 5: *Posterior* summary for θ , ω and p - ZMPL model (zero-inflated case with $\theta = 0.50$).

n	Parameter	Mean	Median	Std. Deviation	95% BCI	
LowerUpper						
Scenario 1						
50	θ	0.6080	0.6160	0.2462	0.2960	1.1175
	ω	0.1664	0.1540	0.0539	0.0805	0.2767
	p	0.2171	0.2291	0.0693	0.1014	0.3793
100	θ	0.5455	0.5535	0.1432	0.3366	0.8333
	ω	0.1651	0.1527	0.0374	0.1005	0.2421
	p	0.2083	0.2203	0.0490	0.1244	0.3130
200	θ	0.5252	0.5332	0.0975	0.3767	0.7103
	ω	0.1635	0.1511	0.0265	0.1162	0.2173
	p	0.2039	0.2159	0.0332	0.1430	0.2750
500	θ	0.5094	0.5174	0.0539	0.4149	0.6176
	ω	0.1624	0.1500	0.0173	0.1315	0.1958
	p	0.2005	0.2125	0.0200	0.1612	0.2437
Scenario 2						
50	θ	0.5274	0.5354	0.0938	0.3792	0.7111
	ω	0.6485	0.6361	0.0693	0.5157	0.7704
	p	0.8096	0.8216	0.0938	0.6331	0.9874
100	θ	0.5133	0.5213	0.0648	0.4083	0.6353
	ω	0.6522	0.6398	0.0458	0.5576	0.7410
	p	0.8071	0.8191	0.0624	0.6822	0.9315
200	θ	0.5074	0.5154	0.0424	0.4323	0.5907
	ω	0.6521	0.6397	0.0316	0.5853	0.7161
	p	0.8041	0.8161	0.0447	0.7160	0.8919
500	θ	0.5037	0.5117	0.0245	0.4557	0.5551
	ω	0.6520	0.6396	0.0224	0.6098	0.6929
	p	0.8020	0.8140	0.0283	0.7463	0.8575

Table 6: *Posterior* summary for θ , ω and p - ZMPL model (zero-inflated case with $\theta = 5.00$).

n	Parameter	Mean	Median	Std. Deviation	95% BCI	
					Lower	Upper
Scenario 1						
50	θ	6.0677	6.0790	3.8594	1.0101	22.0838
	ω	0.0545	0.0683	0.0245	0.0122	0.1287
	p	0.3926	0.3946	0.4136	0.0449	1.5928
100	θ	7.6378	7.6491	5.5890	1.4136	27.0622
	ω	0.0437	0.0575	0.0173	0.0141	0.0899
	p	0.3759	0.3779	0.3758	0.0560	1.4323
200	θ	9.4031	9.4144	8.2113	2.0209	31.7021
	ω	0.0397	0.0535	0.0141	0.0178	0.0701
	p	0.3908	0.3928	0.3886	0.0758	1.3746
500	θ	7.7461	7.7574	6.9236	2.6041	20.7141
	ω	0.0384	0.0522	0.0100	0.0235	0.0567
	p	0.3023	0.3043	0.2449	0.0990	0.8165
Scenario 2						
50	θ	9.0657	9.0770	7.5531	1.9686	30.4054
	ω	0.1550	0.1688	0.0529	0.0726	0.2628
	p	1.4478	1.4498	1.3599	0.2960	4.9892
100	θ	8.6657	8.6770	9.0715	2.5008	25.5350
	ω	0.1536	0.1674	0.0374	0.0912	0.2289
	p	1.3323	1.3343	1.3198	0.3829	3.9335
200	θ	6.8732	6.8845	6.5583	2.8888	15.7468
	ω	0.1521	0.1659	0.0245	0.1063	0.2045
	p	1.0692	1.0712	0.8845	0.4488	2.4440
500	θ	5.5331	5.5444	1.5535	3.3652	8.9952
	ω	0.1512	0.1650	0.0141	0.1213	0.1837
	p	0.8756	0.8776	0.2394	0.5295	1.4302

Table 7: Summary of the Bayesian estimation procedure for zero-inflated samples (ZMPS_h model), $\theta = 0.50$.

n	Parameter	BIAS	VAR	MSE	MAPE (%)
Scenario 1					
50	θ	0.0848	0.0402	0.0474	30.9839
	ω	0.0039	0.0029	0.0029	25.2611
	p	0.0174	0.0045	0.0048	27.0907
100	θ	0.0377	0.0153	0.0167	18.5781
	ω	0.0026	0.0015	0.0015	18.2788
	p	0.0088	0.0022	0.0023	19.1378
200	θ	0.0214	0.0072	0.0076	12.9463
	ω	0.0009	0.0007	0.0007	12.5279
	p	0.0043	0.0011	0.0011	13.1377
500	θ	0.0084	0.0024	0.0025	7.9089
	ω	-0.0003	0.0003	0.0003	7.6647
	p	0.0009	0.0004	0.0004	8.0367
Scenario 2					
50	θ	0.0226	0.0070	0.0076	13.3959
	ω	-0.0039	0.0046	0.0046	7.8656
	p	0.0090	0.0078	0.0079	8.7908
100	θ	0.0107	0.0034	0.0035	9.2027
	ω	-0.0003	0.0021	0.0021	5.3950
	p	0.0062	0.0035	0.0035	6.0171
200	θ	0.0055	0.0015	0.0015	6.0411
	ω	-0.0004	0.0010	0.0010	3.7801
	p	0.0029	0.0018	0.0018	4.3054
500	θ	0.0029	0.0005	0.0005	3.6587
	ω	-0.0001	0.0004	0.0004	2.4419
	p	0.0015	0.0007	0.0007	2.6882

Table 8: Coverage probabilities of the BCIs using zero-inflated samples (ZMPS_h model), $\theta = 0.50$.

n	Parameter	BNCP	CP	ANCP	BNCP	CP	ANCP
		Scenario 1			Scenario 2		
50	θ	3.40	94.00	2.60	4.00	92.80	3.20
	ω	2.60	94.20	3.20	3.80	92.40	3.80
	p	2.40	96.20	1.40	4.20	93.20	2.60
100	θ	3.00	95.00	2.00	3.20	94.40	2.40
	ω	2.40	95.00	2.60	3.20	94.60	2.20
	p	2.80	94.80	2.40	3.00	95.20	1.80
200	θ	2.80	95.00	2.20	3.00	95.40	1.60
	ω	1.60	96.20	2.20	2.20	95.60	2.20
	p	2.40	95.80	1.80	2.60	94.80	2.60
500	θ	2.20	95.80	2.00	1.20	96.00	2.80
	ω	2.00	95.00	3.00	2.40	95.80	1.80
	p	2.60	94.40	3.00	2.80	94.60	2.60

Table 9: Summary of the Bayesian estimation procedure for zero-inflated samples (ZMPS_h model), $\theta = 5.00$.

n	Parameter	BIAS	VAR	MSE	MAPE (%)
Scenario 3					
50	θ	-0.3298	8.3335	8.4422	44.6405
	ω	0.0170	0.0005	0.0008	60.0575
	p	0.1139	0.1030	0.1160	97.1146
100	θ	1.5402	21.4820	23.8543	69.5817
	ω	0.0062	0.0003	0.0003	40.4844
	p	0.1272	0.1014	0.1176	104.3386
200	θ	4.1031	61.3412	78.1763	111.2650
	ω	0.0017	0.0002	0.0002	29.4944
	p	0.1725	0.1358	0.1656	117.6234
500	θ	3.5390	79.4106	91.9354	92.1415
	ω	0.0005	0.0001	0.0001	18.2152
	p	0.1200	0.0869	0.1013	81.7623
Scenario 4					
50	θ	3.6419	48.2332	61.4965	101.3040
	ω	0.0046	0.0025	0.0025	28.8019
	p	0.5587	1.4186	1.7307	100.0529
100	θ	4.2056	94.2202	111.9070	106.9576
	ω	0.0026	0.0012	0.0013	20.3847
	p	0.5749	1.8067	2.1373	95.2501
200	θ	2.2536	43.1892	48.2678	62.2680
	ω	0.0006	0.0006	0.0006	14.0695
	p	0.2952	0.6539	0.7410	54.7191
500	θ	0.6656	3.1470	3.5900	27.0701
	ω	-0.0002	0.0002	0.0002	8.7723
	p	0.0888	0.0688	0.0767	25.0646

Table 10: Coverage probabilities of the BCIs using zero-inflated samples (ZMPS_h model), $\theta = 5.00$.

n	Parameter	BNCP	CP	ANCP	BNCP	CP	ANCP
		Scenario 3			Scenario 4		
50	θ	0.00	89.40	10.60	0.20	94.40	5.40
	ω	4.20	95.80	0.00	3.00	94.40	2.60
	p	2.20	96.80	1.00	2.60	91.60	5.80
100	θ	0.00	92.80	7.20	5.80	91.00	3.20
	ω	3.40	96.60	0.00	3.60	93.00	3.40
	p	0.80	95.00	4.20	5.00	92.00	3.00
200	θ	2.00	92.20	5.80	4.20	93.20	2.60
	ω	1.40	94.20	4.40	2.40	94.40	3.20
	p	3.80	90.80	5.40	4.00	92.20	3.80
500	θ	4.40	92.00	3.60	2.80	94.80	2.40
	ω	2.80	95.20	2.00	2.20	95.80	2.00
	p	3.40	92.20	4.40	2.20	95.40	2.40

Table 11: *Posterior* summary for θ , ω and p - ZMPS_h model (zero-inflated case with $\theta = 0.50$).

n	Parameter	Mean	Median	Std. Deviation	95% BCI	
LowerUpper						
Scenario 1						
50	θ	0.5848	0.5893	0.2005	0.3045	1.0112
	ω	0.1728	0.1633	0.0539	0.0852	0.2846
	p	0.2174	0.2080	0.0671	0.1035	0.3750
100	θ	0.5377	0.5422	0.1237	0.3459	0.7907
	ω	0.1714	0.1619	0.0387	0.1056	0.2496
	p	0.2088	0.1994	0.0469	0.1264	0.3112
200	θ	0.5214	0.5259	0.0849	0.3844	0.6866
	ω	0.1698	0.1603	0.0265	0.1215	0.2242
	p	0.2043	0.1949	0.0332	0.1445	0.2735
500	θ	0.5084	0.5129	0.0490	0.4210	0.6066
	ω	0.1685	0.1590	0.0173	0.1372	0.2025
	p	0.2009	0.1915	0.0200	0.1624	0.2431
Scenario 2						
50	θ	0.5226	0.5271	0.0837	0.3861	0.6870
	ω	0.6717	0.6622	0.0678	0.5401	0.7906
	p	0.8090	0.7996	0.0883	0.6405	0.9767
100	θ	0.5107	0.5152	0.0583	0.4139	0.6210
	ω	0.6753	0.6658	0.0458	0.5820	0.7623
	p	0.8062	0.7968	0.0592	0.6876	0.9235
200	θ	0.5055	0.5100	0.0387	0.4363	0.5813
	ω	0.6752	0.6657	0.0316	0.6093	0.7377
	p	0.8029	0.7935	0.0424	0.7192	0.8857
500	θ	0.5029	0.5074	0.0224	0.4586	0.5499
	ω	0.6754	0.6659	0.0200	0.6339	0.7156
	p	0.8015	0.7921	0.0265	0.7486	0.8539

Table 12: *Posterior* summary for θ , ω and p - ZMPS_h model (zero-inflated case with $\theta = 5.00$).

n	Parameter	Mean	Median	Std. Deviation	95% BCI	
					Lower	Upper
Scenario 1						
50	θ	4.6702	4.6747	2.8868	0.8488	16.9624
	ω	0.0514	0.0419	0.0224	0.0108	0.1240
	p	0.3139	0.3045	0.3209	0.0367	1.2388
100	θ	6.5402	6.5447	4.6349	1.2405	23.4675
	ω	0.0406	0.0311	0.0173	0.0125	0.0855
	p	0.3272	0.3178	0.3184	0.0487	1.2367
200	θ	9.1031	9.1076	7.8321	1.8537	31.7380
	ω	0.0361	0.0266	0.0141	0.0155	0.0654
	p	0.3725	0.3631	0.3685	0.0689	1.3261
500	θ	8.5390	8.5435	8.9113	2.5188	25.0256
	ω	0.0349	0.0254	0.0100	0.0208	0.0526
	p	0.3200	0.3106	0.2948	0.0949	0.9228
Scenario 2						
50	θ	8.6419	8.6464	6.9450	1.7958	29.8891
	ω	0.1422	0.1327	0.0500	0.0637	0.2470
	p	1.3587	1.3493	1.1910	0.2672	4.7205
100	θ	9.2056	9.2101	9.7067	2.4013	28.8794
	ω	0.1403	0.1308	0.0346	0.0808	0.2131
	p	1.3749	1.3655	1.3441	0.3645	4.2288
200	θ	7.2536	7.2581	6.5718	2.7725	17.9926
	ω	0.1382	0.1287	0.0245	0.0944	0.1887
	p	1.0952	1.0858	0.8086	0.4298	2.6290
500	θ	5.6656	5.6701	1.7740	3.2407	9.8288
	ω	0.1374	0.1279	0.0141	0.1088	0.1688
	p	0.8888	0.8794	0.2623	0.5133	1.5163

Table 13: Summary of the Bayesian estimation procedure for zero-inflated samples (ZMPS_u model), $\theta = 0.50$.

n	Parameter	BIAS	VAR	MSE	MAPE (%)
Scenario 1					
50	θ	0.0564	0.0226	0.0258	23.3386
	ω	0.0037	0.0032	0.0032	24.2683
	p	0.0096	0.0038	0.0039	24.6519
100	θ	0.0253	0.0088	0.0094	14.1633
	ω	0.0026	0.0016	0.0016	17.7310
	p	0.0054	0.0019	0.0020	17.9916
200	θ	0.0155	0.0042	0.0045	9.9899
	ω	0.0014	0.0008	0.0008	11.9265
	p	0.0031	0.0009	0.0009	12.0260
500	θ	0.0066	0.0015	0.0016	6.3728
	ω	-0.0001	0.0003	0.0003	7.5151
	p	0.0006	0.0004	0.0004	7.6590
Scenario 2					
50	θ	0.0142	0.0042	0.0044	10.4302
	ω	-0.0065	0.0042	0.0042	6.7847
	p	-0.0015	0.0051	0.0051	7.0153
100	θ	0.0056	0.0020	0.0020	7.0637
	ω	-0.0032	0.0019	0.0019	4.7174
	p	-0.0011	0.0023	0.0023	4.8888
200	θ	0.0034	0.0009	0.0009	4.6752
	ω	-0.0012	0.0010	0.0010	3.3366
	p	0.0001	0.0012	0.0012	3.4981
500	θ	0.0017	0.0003	0.0003	3.0550
	ω	-0.0008	0.0004	0.0004	2.0563
	p	-0.0003	0.0005	0.0005	2.1356

Table 14: Coverage probabilities of the BCIs using zero-inflated samples (ZMPS_u model), $\theta = 0.50$.

n	Parameter	BNCP	CP	ANCP	BNCP	CP	ANCP
		Scenario 1			Scenario 2		
50	θ	3.60	94.00	2.40	4.00	92.80	3.20
	ω	3.80	91.80	4.40	4.60	92.20	3.20
	p	3.80	93.40	2.80	4.60	92.40	3.00
100	θ	3.20	94.60	2.20	3.40	93.60	3.00
	ω	2.40	93.80	3.80	3.00	94.40	2.60
	p	2.60	94.40	3.00	2.80	95.00	2.20
200	θ	2.00	95.80	2.20	2.60	95.80	1.60
	ω	2.40	95.00	2.60	3.60	94.80	1.60
	p	2.80	95.20	2.00	3.20	94.40	2.40
500	θ	2.40	96.00	1.60	1.80	96.40	1.80
	ω	2.40	95.60	2.00	2.60	95.20	2.20
	p	2.60	95.00	2.40	3.00	95.20	1.80

Table 15: Summary of the Bayesian estimation procedure for zero-inflated samples (ZMPS_u model), $\theta = 5.00$.

n	Parameter	BIAS	VAR	MSE	MAPE (%)
Scenario 3					
50	θ	0.8989	11.9023	12.7103	51.0886
	ω	0.0155	0.0006	0.0009	54.7119
	p	0.1682	0.1451	0.1734	116.1277
100	θ	2.3809	26.7883	32.4568	77.7677
	ω	0.0053	0.0004	0.0004	38.2572
	p	0.1652	0.1345	0.1618	116.7277
200	θ	3.7150	56.7867	70.5876	98.3843
	ω	0.0017	0.0002	0.0002	26.4489
	p	0.1726	0.1450	0.1748	113.4977
500	θ	2.0941	37.3556	41.7409	59.9110
	ω	0.0003	0.0001	0.0001	16.4346
	p	0.0863	0.0595	0.0669	63.4104
Scenario 4					
50	θ	3.1828	42.1853	52.3154	87.1116
	ω	0.0035	0.0029	0.0029	26.2223
	p	0.5437	1.4252	1.7207	94.5984
100	θ	2.6997	56.4750	63.7636	72.7466
	ω	0.0023	0.0014	0.0015	18.7618
	p	0.4311	1.3626	1.5485	74.7180
200	θ	1.4198	24.7754	26.7912	42.6647
	ω	0.0008	0.0007	0.0007	12.7431
	p	0.2313	0.5806	0.6341	45.0595
500	θ	0.4012	1.5921	1.7531	18.4887
	ω	-0.0006	0.0003	0.0003	7.8841
	p	0.0638	0.0490	0.0531	20.4783

Table 16: Coverage probabilities of the BCIs using zero-inflated samples (ZMPS_u model), $\theta = 5.00$.

n	Parameter	BNCP	CP	ANCP	BNCP	CP	ANCP
		Scenario 3			Scenario 4		
50	θ	0.00	94.40	5.60	1.60	93.60	4.80
	ω	2.40	97.60	0.00	2.20	94.00	3.80
	p	2.80	96.00	1.20	2.40	93.00	4.60
100	θ	0.00	94.40	5.60	4.40	92.60	3.00
	ω	3.20	96.80	0.00	2.60	94.20	3.20
	p	3.20	93.80	3.00	3.80	92.80	3.40
200	θ	2.40	93.00	4.60	3.40	94.20	2.40
	ω	1.40	94.80	3.80	1.00	96.60	2.40
	p	2.80	93.00	4.20	4.00	92.80	3.20
500	θ	4.00	92.40	3.60	2.60	95.40	2.00
	ω	2.00	95.00	3.00	1.80	95.20	3.00
	p	3.20	92.00	4.80	3.00	94.40	2.60

Table 17: *Posterior* summary for θ , ω and p - ZMPS_u model (zero-inflated case with $\theta = 0.50$).

n	Parameter	Mean	Median	Std. Deviation	95% BCI	
Scenario 1						
50	θ	0.5564	0.5568	0.1503	0.3314	0.8744
	ω	0.1882	0.1984	0.0566	0.0964	0.3030
	p	0.2096	0.2057	0.0616	0.1063	0.3419
100	θ	0.5253	0.5257	0.0938	0.3702	0.7205
	ω	0.1871	0.1973	0.0400	0.1185	0.2675
	p	0.2054	0.2015	0.0436	0.1294	0.2954
200	θ	0.5155	0.5159	0.0648	0.4049	0.6452
	ω	0.1860	0.1962	0.0283	0.1356	0.2421
	p	0.2031	0.1992	0.0300	0.1477	0.2653
500	θ	0.5066	0.5070	0.0387	0.4360	0.5845
	ω	0.1844	0.1946	0.0173	0.1519	0.2194
	p	0.2006	0.1967	0.0200	0.1649	0.2390
Scenario 2						
50	θ	0.5142	0.5146	0.0648	0.4044	0.6426
	ω	0.7315	0.7417	0.0648	0.6051	0.8410
	p	0.7985	0.7946	0.0714	0.6577	0.9247
100	θ	0.5056	0.5060	0.0447	0.4274	0.5928
	ω	0.7349	0.7451	0.0436	0.6455	0.8154
	p	0.7989	0.7950	0.0480	0.6997	0.8902
200	θ	0.5034	0.5038	0.0300	0.4477	0.5637
	ω	0.7369	0.7471	0.0316	0.6743	0.7950
	p	0.8001	0.7962	0.0346	0.7306	0.8655
500	θ	0.5017	0.5021	0.0173	0.4660	0.5392
	ω	0.7372	0.7474	0.0200	0.6979	0.7747
	p	0.7997	0.7958	0.0224	0.7561	0.8417

Table 18: *Posterior* summary for θ , ω and p - ZMPS_u model (zero-inflated case with $\theta = 5.00$).

n	Parameter	Mean	Median	Std. Deviation	95% BCI	
					Lower	Upper
Scenario 1						
50	θ	5.8989	5.9115	3.4500	1.1673	20.6402
	ω	0.0564	0.0501	0.0245	0.0131	0.1315
	p	0.3682	0.3764	0.3809	0.0433	1.4863
100	θ	7.3809	7.3935	5.1757	1.6252	25.1183
	ω	0.0462	0.0399	0.0200	0.0155	0.0934
	p	0.3652	0.3734	0.3667	0.0564	1.3757
200	θ	8.7150	8.7276	7.5357	2.2390	27.9090
	ω	0.0425	0.0362	0.0141	0.0197	0.0739
	p	0.3726	0.3808	0.3808	0.0767	1.2756
500	θ	7.0941	7.1067	6.1119	2.8360	17.3709
	ω	0.0412	0.0349	0.0100	0.0257	0.0601
	p	0.2863	0.2945	0.2439	0.1010	0.7348
Scenario 2						
50	θ	8.1828	8.1954	6.4950	2.1716	25.7631
	ω	0.1669	0.1606	0.0539	0.0809	0.2773
	p	1.3437	1.3519	1.1938	0.2979	4.4706
100	θ	7.6997	7.7123	7.5150	2.7085	20.7617
	ω	0.1657	0.1594	0.0374	0.1010	0.2430
	p	1.2311	1.2393	1.1673	0.3875	3.4284
200	θ	6.4198	6.4324	4.9775	3.1480	13.2801
	ω	0.1642	0.1579	0.0265	0.1167	0.2180
	p	1.0313	1.0395	0.7620	0.4647	2.2119
500	θ	5.4012	5.4138	1.2618	3.5952	8.1502
	ω	0.1629	0.1566	0.0173	0.1320	0.1963
	p	0.8638	0.8720	0.2214	0.5439	1.3539

1.1.2 Zero-deflated artificial data

The following tables illustrate the results obtained in the first simulation study, considering scenarios 1-4 of the zero-deflated case as described in Section 6 (Table 7) of the paper.

Table 19: Summary of the Bayesian estimation procedure for zero-deflated samples (ZMPL model), $\theta = 2.50$.

n	Parameter	BIAS	VAR	MSE	MAPE (%)
Scenario 1					
50	θ	0.5872	2.2849	2.6296	38.7108
	ω	-0.0036	0.0056	0.0056	12.3448
	p	0.2543	0.4621	0.5268	33.6599
100	θ	0.2517	0.5681	0.6315	22.0659
	ω	0.0009	0.0027	0.0027	8.6409
	p	0.1165	0.1219	0.1355	19.5120
200	θ	0.1320	0.2202	0.2376	14.6123
	ω	0.0009	0.0012	0.0012	5.7611
	p	0.0646	0.0541	0.0583	13.3916
500	θ	0.0615	0.0700	0.0738	8.5006
	ω	0.0006	0.0005	0.0005	3.7450
	p	0.0305	0.0197	0.0206	8.0638
Scenario 2					
50	θ	0.2958	0.6334	0.7209	24.1878
	ω	-0.0059	0.0033	0.0034	5.5602
	p	0.2167	0.3921	0.4391	19.8135
100	θ	0.1347	0.2598	0.2780	15.3883
	ω	-0.0030	0.0013	0.0013	3.4556
	p	0.0979	0.1568	0.1664	12.4787
200	θ	0.0697	0.1086	0.1134	9.9585
	ω	-0.0020	0.0008	0.0008	2.6564
	p	0.0498	0.0706	0.0731	8.4424
500	θ	0.0298	0.0367	0.0376	6.1903
	ω	-0.0016	0.0003	0.0003	1.7113
	p	0.0191	0.0241	0.0245	5.2450

Table 20: Coverage probabilities of the BCIs using zero-deflated samples (ZMPL model), $\theta = 2.50$.

n	Parameter	BNCP	CP	ANCP	BNCP	CP	ANCP
		Scenario 1			Scenario 2		
50	θ	4.80	91.80	3.40	3.40	93.40	3.20
	ω	3.20	92.40	4.40	3.20	94.00	2.80
	p	4.00	92.20	3.80	3.60	93.60	2.80
100	θ	3.20	94.20	2.60	2.60	95.00	2.40
	ω	4.00	93.60	2.40	2.00	95.80	2.20
	p	3.60	93.60	2.80	3.00	94.40	2.60
200	θ	4.00	93.80	2.20	3.20	95.00	1.80
	ω	2.00	95.60	2.40	2.60	94.40	3.00
	p	3.60	93.20	3.20	2.80	94.40	2.80
500	θ	3.80	94.00	2.20	2.00	94.80	3.20
	ω	1.60	95.80	2.60	1.80	94.80	3.40
	p	4.40	93.00	2.60	2.60	94.20	3.20

Table 21: Summary of the Bayesian estimation procedure for zero-deflated samples (ZMPL model), $\theta = 6.00$.

n	Parameter	BIAS	VAR	MSE	MAPE (%)
Scenario 3					
50	θ	5.1449	104.0101	130.4798	110.0221
	ω	0.0024	0.0038	0.0038	21.9213
	p	1.1601	6.0880	7.4338	108.0458
100	θ	4.0243	124.7600	140.9553	87.1485
	ω	0.0022	0.0020	0.0020	15.8825
	p	0.8806	5.7641	6.5396	83.5732
200	θ	1.9601	51.2473	55.0892	48.1096
	ω	0.0008	0.0009	0.0009	10.9028
	p	0.4331	2.3479	2.5355	46.7987
500	θ	0.5535	2.8691	3.1754	20.6722
	ω	-0.0004	0.0004	0.0004	6.7408
	p	0.1217	0.1561	0.1709	21.2245
Scenario 4					
50	θ	4.0865	105.3742	122.0737	89.6096
	ω	0.0015	0.0051	0.0051	14.7628
	p	1.5751	17.0394	19.5203	86.5442
100	θ	1.7836	47.6696	50.8508	46.6722
	ω	0.0023	0.0028	0.0028	10.8051
	p	0.7057	8.4348	8.9327	46.1133
200	θ	0.8054	6.4923	7.1410	27.1256
	ω	-0.0001	0.0012	0.0012	7.0980
	p	0.3077	0.8990	0.9937	27.0022
500	θ	0.3258	1.2229	1.3290	14.6293
	ω	-0.0001	0.0005	0.0005	4.5344
	p	0.1257	0.1985	0.2143	14.7487

Table 22: Coverage probabilities of the BCIs using zero-deflated samples (ZMPL model), $\theta = 6.00$.

n	Parameter	BNCP	CP	ANCP	BNCP	CP	ANCP
		Scenario 3			Scenario 4		
50	θ	4.80	90.60	4.60	3.60	92.40	4.00
	ω	3.20	93.40	3.40	3.40	93.00	3.60
	p	4.60	91.20	4.20	4.40	91.20	4.40
100	θ	5.60	90.80	3.60	3.20	92.80	4.00
	ω	3.40	92.60	4.00	4.60	91.20	4.20
	p	5.40	90.60	4.00	3.00	94.00	3.00
200	θ	4.00	93.40	2.60	2.40	93.80	3.80
	ω	2.20	94.60	3.20	2.40	94.40	3.20
	p	4.80	92.40	2.80	2.60	93.20	4.20
500	θ	3.00	95.20	1.80	2.20	94.60	3.20
	ω	4.20	92.40	3.40	2.00	94.80	3.20
	p	2.60	94.60	2.80	4.00	92.60	3.40

Table 23: *Posterior* summary for θ , ω and p - ZMPL model (zero-deflated case with $\theta = 2.50$).

n	Parameter	Mean	Median	Std. Deviation	95% BCI	
LowerUpper						
Scenario 1						
50	θ	3.0872	3.1010	1.5116	1.5622	5.8608
	ω	0.4780	0.4798	0.0748	0.3453	0.6123
	p	1.6543	1.6465	0.6798	0.8902	3.0310
100	θ	2.7517	2.7655	0.7537	1.7443	4.2541
	ω	0.4825	0.4843	0.0520	0.3868	0.5790
	p	1.5165	1.5087	0.3491	1.0023	2.2782
200	θ	2.6320	2.6458	0.4693	1.9193	3.5687
	ω	0.4826	0.4844	0.0346	0.4141	0.5514
	p	1.4646	1.4568	0.2326	1.0988	1.9444
500	θ	2.5615	2.5753	0.2646	2.1044	3.1041
	ω	0.4822	0.4840	0.0224	0.4386	0.5259
	p	1.4305	1.4227	0.1404	1.1956	1.7090
Scenario 2						
50	θ	2.7958	2.8096	0.7959	1.7079	4.4682
	ω	0.8198	0.8216	0.0574	0.7067	0.9092
	p	2.6167	2.6089	0.6262	1.7507	3.9608
100	θ	2.6347	2.6485	0.5097	1.8749	3.6565
	ω	0.8227	0.8245	0.0361	0.7436	0.8898
	p	2.4979	2.4901	0.3960	1.8921	3.3185
200	θ	2.5697	2.5835	0.3295	2.0261	3.2393
	ω	0.8236	0.8254	0.0283	0.7685	0.8727
	p	2.4498	2.4420	0.2657	2.0163	2.9869
500	θ	2.5298	2.5436	0.1916	2.1795	2.9285
	ω	0.8241	0.8259	0.0173	0.7897	0.8560
	p	2.4191	2.4113	0.1552	2.1396	2.7389

Table 24: *Posterior* summary for θ , ω and p - ZMPL model (zero-deflated case with $\theta = 6.00$).

n	Parameter	Mean	Median	Std. Deviation	95% BCI	
					Lower	Upper
Scenario 1						
50	θ	11.1449	11.1514	10.1985	2.6134	36.4116
	ω	0.2269	0.2156	0.0616	0.1260	0.3481
	p	2.5601	2.5554	2.4674	0.5852	8.4908
100	θ	10.0243	10.0308	11.1696	3.2102	27.9620
	ω	0.2267	0.2154	0.0447	0.1515	0.3121
	p	2.2806	2.2759	2.4009	0.7325	6.3537
200	θ	7.9601	7.9666	7.1587	3.6697	16.9878
	ω	0.2253	0.2140	0.0300	0.1707	0.2851
	p	1.8331	1.8284	1.5323	0.8456	3.9126
500	θ	6.5535	6.5600	1.6938	4.2039	10.1489
	ω	0.2241	0.2128	0.0200	0.1888	0.2614
	p	1.5217	1.5170	0.3951	0.9725	2.3654
Scenario 2						
50	θ	10.0865	10.0930	10.2652	3.0750	28.8456
	ω	0.3863	0.3750	0.0714	0.2605	0.5207
	p	3.9751	3.9704	4.1279	1.2230	11.3772
100	θ	7.7836	7.7901	6.9043	3.4684	16.9435
	ω	0.3871	0.3758	0.0529	0.2956	0.4830
	p	3.1057	3.1010	2.9043	1.3970	6.7629
200	θ	6.8054	6.8119	2.5480	3.9472	11.6039
	ω	0.3847	0.3734	0.0346	0.3190	0.4525
	p	2.7077	2.7030	0.9482	1.5792	4.6128
500	θ	6.3258	6.3323	1.1058	4.5316	8.7902
	ω	0.3848	0.3735	0.0224	0.3427	0.4276
	p	2.5257	2.5210	0.4455	1.8143	3.5074

Table 25: Summary of the Bayesian estimation procedure for zero-deflated samples (ZMPS_h model), $\theta = 2.50$.

n	Parameter	BIAS	VAR	MSE	MAPE (%)
Scenario 1					
50	θ	0.6431	2.6288	3.0424	40.7919
	ω	-0.0026	0.0052	0.0052	13.0551
	p	0.2704	0.5382	0.6113	35.5518
100	θ	0.2778	0.6263	0.7034	22.7574
	ω	0.0008	0.0026	0.0026	9.2075
	p	0.1246	0.1419	0.1574	20.8770
200	θ	0.1470	0.2489	0.2705	14.9533
	ω	0.0003	0.0012	0.0012	6.2788
	p	0.0685	0.0649	0.0696	14.4486
500	θ	0.0630	0.0794	0.0833	8.9057
	ω	0.0001	0.0005	0.0005	3.9387
	p	0.0299	0.0237	0.0246	8.6947
Scenario 2					
50	θ	0.3842	1.1667	1.3143	28.1364
	ω	-0.0061	0.0040	0.0041	6.5609
	p	0.2752	0.7434	0.8192	23.9563
100	θ	0.1623	0.3541	0.3805	16.8785
	ω	-0.0037	0.0018	0.0018	4.5426
	p	0.1136	0.2357	0.2486	14.4426
200	θ	0.0774	0.1230	0.1290	10.8767
	ω	-0.0017	0.0010	0.0010	3.2379
	p	0.0557	0.0912	0.0943	9.8702
500	θ	0.0353	0.0398	0.0410	6.3614
	ω	-0.0012	0.0003	0.0004	1.9701
	p	0.0241	0.0301	0.0307	5.7739

Table 26: Coverage probabilities of the BCIs using zero-deflated samples (ZMPS_h model), $\theta = 2.50$.

n	Parameter	BNCP	CP	ANCP	BNCP	CP	ANCP
		Scenario 1			Scenario 2		
50	θ	3.80	92.60	3.60	4.20	92.60	3.20
	ω	4.20	91.60	4.20	3.40	93.00	3.60
	p	2.40	94.20	3.40	3.60	93.60	2.80
100	θ	2.60	95.00	2.40	3.60	93.60	2.80
	ω	3.20	94.60	2.20	2.80	95.20	2.00
	p	2.40	95.40	2.20	3.20	94.00	2.80
200	θ	3.80	93.00	3.20	3.20	94.40	2.40
	ω	1.40	96.20	2.40	3.60	94.00	2.40
	p	3.20	93.60	3.20	3.00	94.00	3.00
500	θ	3.40	94.60	2.00	2.80	94.40	2.80
	ω	1.60	96.20	2.20	2.40	95.20	2.40
	p	3.80	93.00	3.20	2.40	94.60	3.00

Table 27: Summary of the Bayesian estimation procedure for zero-deflated samples (ZMPS_h model), $\theta = 6.00$.

n	Parameter	BIAS	VAR	MSE	MAPE (%)
Scenario 3					
50	θ	5.2632	103.0505	130.7522	114.6161
	ω	0.0025	0.0036	0.0036	23.4158
	p	1.1039	5.4860	6.7046	107.4200
100	θ	4.7599	160.4740	183.1310	100.7584
	ω	0.0019	0.0018	0.0019	16.7770
	p	0.9399	6.4959	7.3793	88.7662
200	θ	2.4685	84.6359	90.7293	57.1014
	ω	0.0007	0.0009	0.0009	11.4027
	p	0.4850	2.9058	3.1411	50.6630
500	θ	0.6439	3.6796	4.0943	22.8275
	ω	-0.0003	0.0003	0.0003	7.0875
	p	0.1288	0.1732	0.1898	22.0107
Scenario 4					
50	θ	5.6309	174.5981	206.3046	116.8171
	ω	0.0018	0.0049	0.0049	15.8665
	p	1.9876	24.0226	27.9730	105.9069
100	θ	2.4184	69.2322	75.0807	58.5291
	ω	0.0021	0.0026	0.0026	11.3873
	p	0.8590	9.4644	10.2023	53.7601
200	θ	1.1560	13.2861	14.6224	33.6414
	ω	0.0004	0.0012	0.0012	7.7537
	p	0.4053	1.5395	1.7037	31.2498
500	θ	0.4167	1.7525	1.9261	16.9115
	ω	0.0000	0.0005	0.0005	4.8717
	p	0.1482	0.2586	0.2806	16.3072

Table 28: Coverage probabilities of the BCIs using zero-deflated samples (ZMPS_h model), $\theta = 6.00$.

n	Parameter	BNCP	CP	ANCP	BNCP	CP	ANCP
		Scenario 3			Scenario 4		
50	θ	2.60	93.40	4.00	7.20	87.20	5.60
	ω	3.20	92.80	4.00	3.40	92.40	4.20
	p	3.80	91.40	4.80	6.20	89.20	4.60
100	θ	5.60	91.60	2.80	3.80	92.60	3.60
	ω	3.40	93.40	3.20	4.40	92.80	2.80
	p	4.80	92.20	3.00	3.60	92.80	3.60
200	θ	3.40	94.00	2.60	4.20	91.60	4.20
	ω	3.00	94.20	2.80	3.00	93.80	3.20
	p	4.20	93.20	2.60	4.00	91.80	4.20
500	θ	2.20	95.40	2.40	3.00	93.80	3.20
	ω	3.40	93.60	3.00	2.60	94.40	3.00
	p	2.00	95.80	2.20	2.80	93.20	4.00

Table 29: *Posterior* summary for θ , ω and p - ZMPS_h model (zero-deflated case with $\theta = 2.50$).

n	Parameter	Mean	Median	Std. Deviation	95% BCI	
Scenario 1						
50	θ	3.1431	3.1551	1.6214	1.5440	6.3161
	ω	0.4368	0.4451	0.0721	0.3066	0.5719
	p	1.6704	1.6608	0.7336	0.8400	3.2247
100	θ	2.7778	2.7898	0.7914	1.7307	4.4493
	ω	0.4402	0.4485	0.0510	0.3457	0.5368
	p	1.5246	1.5150	0.3767	0.9612	2.3854
200	θ	2.6470	2.6590	0.4989	1.9081	3.6658
	ω	0.4397	0.4480	0.0346	0.3721	0.5084
	p	1.4685	1.4589	0.2548	1.0660	2.0069
500	θ	2.5630	2.5750	0.2818	2.0912	3.1394
	ω	0.4395	0.4478	0.0224	0.3965	0.4831
	p	1.4299	1.4203	0.1539	1.1718	1.7398
Scenario 2						
50	θ	2.8842	2.8962	1.0801	1.7100	4.8506
	ω	0.7472	0.7555	0.0632	0.6227	0.8536
	p	2.6752	2.6656	0.8622	1.6777	4.2827
100	θ	2.6623	2.6743	0.5951	1.8679	3.7900
	ω	0.7496	0.7579	0.0424	0.6619	0.8280
	p	2.5136	2.5040	0.4855	1.8282	3.4597
200	θ	2.5774	2.5894	0.3507	2.0153	3.2941
	ω	0.7515	0.7598	0.0316	0.6900	0.8084
	p	2.4557	2.4461	0.3020	1.9673	3.0660
500	θ	2.5353	2.5473	0.1995	2.1727	2.9569
	ω	0.7520	0.7603	0.0173	0.7134	0.7887
	p	2.4241	2.4145	0.1735	2.1088	2.7865

Table 30: *Posterior* summary for θ , ω and p - ZMPS_h model (zero-deflated case with $\theta = 6.00$).

n	Parameter	Mean	Median	Std. Deviation	95% BCI	
					Lower	Upper
Scenario 1						
50	θ	11.2632	11.2640	10.1514	2.4288	38.3828
	ω	0.2072	0.2148	0.0600	0.1107	0.3253
	p	2.5039	2.4977	2.3422	0.5412	8.4682
100	θ	10.7599	10.7607	12.6678	3.0569	32.2883
	ω	0.2066	0.2142	0.0424	0.1345	0.2896
	p	2.3399	2.3337	2.5487	0.6935	6.8301
200	θ	8.4685	8.4693	9.1998	3.5072	19.8132
	ω	0.2054	0.2130	0.0300	0.1528	0.2634
	p	1.8850	1.8788	1.7046	0.8107	4.2752
500	θ	6.6439	6.6447	1.9182	4.0268	10.8969
	ω	0.2043	0.2119	0.0173	0.1703	0.2406
	p	1.5288	1.5226	0.4162	0.9415	2.4659
Scenario 2						
50	θ	11.6309	11.6317	13.2136	3.0316	36.4990
	ω	0.3526	0.3602	0.0700	0.2305	0.4856
	p	4.3876	4.3814	4.9013	1.1980	13.4623
100	θ	8.4184	8.4192	8.3206	3.3456	20.1552
	ω	0.3529	0.3605	0.0510	0.2637	0.4475
	p	3.2590	3.2528	3.0764	1.3496	7.5749
200	θ	7.1560	7.1568	3.6450	3.8282	13.2586
	ω	0.3512	0.3588	0.0346	0.2870	0.4181
	p	2.8053	2.7991	1.2408	1.5454	5.0653
500	θ	6.4167	6.4175	1.3238	4.3964	9.3340
	ω	0.3508	0.3584	0.0224	0.3098	0.3929
	p	2.5482	2.5420	0.5085	1.7765	3.6462

Table 31: Summary of the Bayesian estimation procedure for zero-deflated samples (ZMPS_u model), $\theta = 2.50$.

n	Parameter	BIAS	VAR	MSE	MAPE (%)
Scenario 1					
50	θ	0.3166	0.7945	0.8947	24.9405
	ω	-0.0030	0.0055	0.0055	10.4541
	p	0.1658	0.2344	0.2619	25.0004
100	θ	0.1468	0.2801	0.3016	15.7823
	ω	0.0007	0.0025	0.0025	7.0281
	p	0.0823	0.0836	0.0903	15.5576
200	θ	0.0784	0.1070	0.1131	10.2906
	ω	0.0008	0.0011	0.0011	4.8188
	p	0.0457	0.0337	0.0358	10.5361
500	θ	0.0392	0.0365	0.0380	6.0358
	ω	0.0006	0.0005	0.0005	3.0695
	p	0.0229	0.0129	0.0135	6.4015
Scenario 2					
50	θ	0.1774	0.3151	0.3466	17.2343
	ω	-0.0068	0.0009	0.0009	2.4583
	p	0.1512	0.2399	0.2627	15.4492
100	θ	0.0781	0.1335	0.1396	11.0977
	ω	-0.0030	0.0004	0.0004	1.6877
	p	0.0675	0.0997	0.1043	9.9036
200	θ	0.0440	0.0520	0.0539	7.0624
	ω	-0.0014	0.0002	0.0002	1.2590
	p	0.0382	0.0405	0.0419	6.4454
500	θ	0.0209	0.0189	0.0193	4.4106
	ω	-0.0002	0.0001	0.0001	0.7784
	p	0.0189	0.0147	0.0150	4.0805

Table 32: Coverage probabilities of the BCIs using zero-deflated samples (ZMPS_u model), $\theta = 2.50$.

n	Parameter	BNCP	CP	ANCP	BNCP	CP	ANCP
		Scenario 1			Scenario 2		
50	θ	4.00	92.80	3.20	4.00	92.60	3.40
	ω	3.40	93.20	3.40	0.00	97.60	2.40
	p	3.60	92.80	3.60	3.80	93.20	3.00
100	θ	2.80	95.00	2.20	3.20	93.80	3.00
	ω	3.80	94.80	1.40	3.00	94.00	3.00
	p	2.60	95.60	1.80	3.20	94.00	2.80
200	θ	3.40	93.40	3.20	3.60	94.00	2.40
	ω	1.80	96.80	1.40	2.60	94.80	2.60
	p	3.00	93.20	3.80	3.20	94.60	2.20
500	θ	3.80	92.60	3.60	2.20	95.40	2.40
	ω	3.20	94.00	2.80	3.40	94.40	2.20
	p	3.60	93.20	3.20	2.80	95.00	2.20

Table 33: Summary of the Bayesian estimation procedure for zero-deflated samples (ZMPS_u model), $\theta = 6.00$.

n	Parameter	BIAS	VAR	MSE	MAPE (%)
Scenario 3					
50	θ	4.4422	88.5931	108.3263	96.1490
	ω	0.0026	0.0039	0.0039	21.1509
	p	1.0729	5.7678	6.9189	101.9241
100	θ	2.9044	76.8044	85.2397	66.2762
	ω	0.0017	0.0020	0.0020	14.8816
	p	0.6840	3.8959	4.3638	69.7217
200	θ	1.2096	11.5184	12.9814	34.0224
	ω	0.0011	0.0009	0.0009	10.2884
	p	0.2959	0.6304	0.7180	36.7814
500	θ	0.4511	2.0118	2.2154	17.3364
	ω	-0.0001	0.0004	0.0004	6.4957
	p	0.1097	0.1329	0.1449	19.4601
Scenario 4					
50	θ	3.6216	95.9068	109.0225	79.9508
	ω	-0.0003	0.0052	0.0052	14.0394
	p	1.4694	17.2267	19.3858	82.0812
100	θ	1.4939	48.8965	51.1283	40.4640
	ω	0.0017	0.0028	0.0028	10.2507
	p	0.6362	10.4698	10.8746	42.5884
200	θ	0.6921	4.7636	5.2426	23.4989
	ω	0.0002	0.0012	0.0012	6.7406
	p	0.2859	0.7846	0.8663	25.2916
500	θ	0.2740	0.9269	1.0020	12.6203
	ω	-0.0002	0.0005	0.0005	4.2638
	p	0.1139	0.1801	0.1931	13.8640

Table 34: Coverage probabilities of the BCIs using zero-deflated samples (ZMPS_u model), $\theta = 6.00$.

n	Parameter	BNCP	CP	ANCP	BNCP	CP	ANCP
		Scenario 3			Scenario 4		
50	θ	5.00	90.00	5.00	4.00	92.80	3.20
	ω	3.60	92.80	3.60	3.00	93.80	3.20
	p	3.80	91.80	4.40	4.80	91.40	3.80
100	θ	4.40	91.60	4.00	2.40	94.20	3.40
	ω	3.20	92.80	4.00	3.60	93.20	3.20
	p	4.00	91.60	4.40	2.40	93.80	3.80
200	θ	3.60	93.80	2.60	2.80	94.00	3.20
	ω	2.80	94.60	2.60	2.20	95.60	2.20
	p	3.80	92.40	3.80	3.60	92.80	3.60
500	θ	2.80	95.40	1.80	2.60	95.00	2.40
	ω	3.40	93.80	2.80	1.60	94.80	3.60
	p	3.00	94.20	2.80	2.00	95.00	3.00

Table 35: Posterior summary for θ , ω and p - ZMPS_u model (zero-deflated case with $\theta = 2.50$).

n	Parameter	Mean	Median	Std. Deviation	95% BCI	
LowerUpper						
Scenario 1						
50	θ	2.8166	2.8131	0.8913	1.7253	4.5418
	ω	0.5546	0.5560	0.0742	0.4198	0.6854
	p	1.5658	1.5543	0.4841	0.9405	2.5715
100	θ	2.6468	2.6433	0.5292	1.8972	3.6660
	ω	0.5583	0.5597	0.0500	0.4617	0.6527
	p	1.4823	1.4708	0.2891	1.0479	2.0817
200	θ	2.5784	2.5749	0.3271	2.0453	3.2381
	ω	0.5584	0.5598	0.0332	0.4897	0.6260
	p	1.4457	1.4342	0.1836	1.1359	1.8331
500	θ	2.5392	2.5357	0.1910	2.1965	2.9302
	ω	0.5582	0.5596	0.0224	0.5145	0.6013
	p	1.4229	1.4114	0.1136	1.2232	1.6527
Scenario 2						
50	θ	2.6774	2.6739	0.5613	1.8661	3.8085
	ω	0.9490	0.9504	0.0300	0.8785	0.9882
	p	2.5512	2.5397	0.4898	1.8554	3.5666
100	θ	2.5781	2.5746	0.3654	2.0070	3.2954
	ω	0.9529	0.9543	0.0200	0.9054	0.9838
	p	2.4675	2.4560	0.3158	1.9772	3.1053
200	θ	2.5440	2.5405	0.2280	2.1346	3.0244
	ω	0.9545	0.9559	0.0141	0.9222	0.9782
	p	2.4382	2.4267	0.2012	2.0858	2.8626
500	θ	2.5209	2.5174	0.1375	2.2581	2.8118
	ω	0.9556	0.9570	0.0100	0.9360	0.9717
	p	2.4189	2.4074	0.1212	2.1917	2.6744

Table 36: *Posterior* summary for θ , ω and p - ZMPS_u model (zero-deflated case with $\theta = 6.00$).

n	Parameter	Mean	Median	Std. Deviation	95% BCI	
					Lower	Upper
Scenario 1						
50	θ	10.4422	10.4448	9.4124	2.8623	32.5257
	ω	0.2404	0.2512	0.0624	0.1365	0.3635
	p	2.4729	2.4853	2.4016	0.5952	8.0047
100	θ	8.9044	8.9070	8.7638	3.4017	22.6765
	ω	0.2396	0.2504	0.0447	0.1626	0.3264
	p	2.0840	2.0964	1.9738	0.7321	5.4541
200	θ	7.2096	7.2122	3.3939	3.8561	13.6095
	ω	0.2389	0.2497	0.0300	0.1829	0.2999
	p	1.6959	1.7083	0.7940	0.8501	3.3083
500	θ	6.4511	6.4537	1.4184	4.4318	9.4385
	ω	0.2377	0.2485	0.0200	0.2016	0.2757
	p	1.5097	1.5221	0.3646	0.9950	2.2739
Scenario 2						
50	θ	9.6216	9.6242	9.7932	3.3481	26.1363
	ω	0.4074	0.4182	0.0721	0.2795	0.5422
	p	3.8694	3.8818	4.1505	1.2454	10.7738
100	θ	7.4939	7.4965	6.9926	3.7456	15.2235
	ω	0.4094	0.4202	0.0529	0.3166	0.5058
	p	3.0362	3.0486	3.2357	1.4294	6.3587
200	θ	6.6921	6.6947	2.1826	4.2119	10.7335
	ω	0.4079	0.4187	0.0346	0.3414	0.4762
	p	2.6859	2.6983	0.8858	1.6246	4.4158
500	θ	6.2740	6.2766	0.9628	4.7273	8.3450
	ω	0.4076	0.4184	0.0224	0.3651	0.4508
	p	2.5139	2.5263	0.4244	1.8472	3.4110

1.2 Study 2

The following tables illustrate the results obtained in the second simulation study, considering scenarios 1-4 of zero-inflated and zero-deflated cases as described in Section 6 (Table 7) of the paper.

Table 37: Estimates of correct selection probability when ZMPL is the true model (zero-inflation).

Scenario	n	Fitted	% DIC	% EAIC	% EBIC	% LMPL
1	50	ZMPL	64.20	64.40	64.40	70.60
		ZMP	35.80	35.60	35.60	29.40
	100	ZMPL	82.60	82.60	82.60	85.80
		ZMP	17.40	17.40	17.40	14.20
	200	ZMPL	94.80	94.80	94.80	95.80
		ZMP	5.20	5.20	5.20	4.20
	500	ZMPL	99.80	99.80	99.80	100.00
		ZMP	0.20	0.20	0.20	0.00
2	50	ZMPL	95.80	95.80	95.80	96.20
		ZMP	4.20	4.20	4.20	3.80
	100	ZMPL	99.20	99.20	99.20	99.20
		ZMP	0.80	0.80	0.80	0.80
	200	ZMPL	100.00	100.00	100.00	100.00
		ZMP	0.00	0.00	0.00	0.00
	500	ZMPL	100.00	100.00	100.00	100.00
		ZMP	0.00	0.00	0.00	0.00
3	50	ZMPL	72.00	70.00	70.00	86.20
		ZMP	28.00	30.00	30.00	13.80
	100	ZMPL	60.40	54.00	54.00	74.20
		ZMP	39.60	46.00	46.00	25.80
	200	ZMPL	50.40	39.80	39.80	57.20
		ZMP	49.60	60.20	60.20	42.80
	500	ZMPL	37.40	34.00	34.00	43.00
		ZMP	62.60	66.00	66.00	57.00
4	50	ZMPL	47.00	38.00	38.00	57.40
		ZMP	53.00	62.00	62.00	42.60
	100	ZMPL	37.20	33.20	33.20	43.40
		ZMP	62.80	66.80	66.80	56.60
	200	ZMPL	42.20	40.80	40.80	49.00
		ZMP	57.80	59.20	59.20	51.00
	500	ZMPL	51.80	52.00	52.00	57.00
		ZMP	48.20	48.00	48.00	43.00

Table 38: Estimates of correct selection probability when ZMPL is the true model (zero-deflation).

Scenario	n	Fitted	% DIC	% EAIC	% EBIC	% LMPL
1	50	ZMPL	49.40	49.20	49.20	56.60
		ZMP	50.60	50.80	50.80	43.40
	100	ZMPL	62.20	62.00	62.00	65.80
		ZMP	37.80	38.00	38.00	34.20
	200	ZMPL	71.60	71.40	71.40	74.00
		ZMP	28.40	28.40	28.40	26.00
	500	ZMPL	85.80	86.20	86.20	86.80
		ZMP	14.20	13.80	13.80	13.20
2	50	ZMPL	58.60	58.80	58.80	63.00
		ZMP	41.40	41.20	41.20	37.00
	100	ZMPL	68.00	68.00	68.00	70.60
		ZMP	32.00	32.00	32.00	29.40
	200	ZMPL	80.20	80.20	80.20	81.80
		ZMP	19.80	19.80	19.80	18.20
	500	ZMPL	92.40	92.40	92.40	92.80
		ZMP	7.60	7.60	7.60	7.20
3	50	ZMPL	40.80	33.80	33.80	47.40
		ZMP	59.20	66.20	66.20	52.60
	100	ZMPL	34.80	32.00	32.00	39.60
		ZMP	65.20	68.00	68.00	60.40
	200	ZMPL	45.20	44.20	44.20	49.80
		ZMP	54.80	55.80	55.80	50.20
	500	ZMPL	51.60	52.40	52.40	55.40
		ZMP	48.40	47.60	47.60	44.60
4	50	ZMPL	37.00	33.20	33.20	43.60
		ZMP	63.00	66.80	66.80	56.40
	100	ZMPL	40.60	40.00	40.00	45.80
		ZMP	59.40	60.00	60.00	54.20
	200	ZMPL	48.20	47.80	47.80	53.00
		ZMP	51.80	52.20	52.20	47.00
	500	ZMPL	59.60	59.60	59.60	62.00
		ZMP	40.40	40.40	40.40	38.00

Table 39: Estimates of correct selection probability when ZMPS_h is the true model (zero-inflation).

Scenario	n	Fitted	% DIC	% EAIC	% EBIC	% LMPL
1	50	ZMPS_h	67.60	67.20	67.20	75.00
		ZMP	32.40	32.80	32.80	25.00
	100	ZMPS_h	85.00	84.80	84.80	87.60
		ZMP	15.00	15.20	15.20	12.40
	200	ZMPS_h	94.60	94.60	94.60	95.60
		ZMP	5.40	5.40	5.40	4.40
	500	ZMPS_h	100.00	100.00	100.00	100.00
		ZMP	0.00	0.00	0.00	0.00
2	50	ZMPS_h	94.80	94.80	94.80	96.20
		ZMP	5.20	5.20	5.20	3.80
	100	ZMPS_h	99.20	99.20	99.20	99.40
		ZMP	0.80	0.80	0.80	0.60
	200	ZMPS_h	100.00	100.00	100.00	100.00
		ZMP	0.00	0.00	0.00	0.00
	500	ZMPS_h	100.00	100.00	100.00	100.00
		ZMP	0.00	0.00	0.00	0.00
3	50	ZMPS_h	73.60	71.40	71.40	83.00
		ZMP	26.40	28.60	28.60	17.00
	100	ZMPS_h	61.60	56.20	56.20	75.40
		ZMP	38.40	43.80	43.80	24.60
	200	ZMPS_h	45.80	40.20	40.20	55.40
		ZMP	54.20	59.80	59.80	44.60
	500	ZMPS_h	35.00	31.40	31.40	41.60
		ZMP	65.00	68.60	68.60	58.40
4	50	ZMPS_h	42.40	36.60	36.60	54.80
		ZMP	57.60	63.40	63.40	45.20
	100	ZMPS_h	34.00	31.60	31.60	44.80
		ZMP	66.00	68.40	68.40	55.20
	200	ZMPS_h	37.80	36.80	36.80	45.60
		ZMP	62.20	63.20	63.20	54.40
	500	ZMPS_h	52.20	52.40	52.40	56.60
		ZMP	47.80	47.60	47.60	43.40

Table 40: Estimates of correct selection probability when ZMPS_h is the true model (zero-deflation).

Scenario	n	Fitted	% DIC	% EAIC	% EBIC	% LMPL
1	50	ZMPS_h	45.80	45.40	45.40	52.80
		ZMP	54.20	54.60	54.60	47.20
	100	ZMPS_h	58.80	59.20	59.20	64.00
		ZMP	41.20	40.80	40.80	36.00
	200	ZMPS_h	71.00	71.00	71.00	73.40
		ZMP	29.00	29.00	29.00	26.60
	500	ZMPS_h	86.20	86.40	86.40	87.40
		ZMP	13.80	13.60	13.60	12.60
2	50	ZMPS_h	55.00	55.00	55.00	60.20
		ZMP	45.00	45.00	45.00	39.80
	100	ZMPS_h	66.60	66.60	66.60	70.00
		ZMP	33.40	33.40	33.40	30.00
	200	ZMPS_h	77.20	77.60	77.60	79.20
		ZMP	22.80	22.40	22.40	20.80
	500	ZMPS_h	90.20	90.00	90.00	91.00
		ZMP	9.80	10.00	10.00	9.00
3	50	ZMPS_h	36.40	31.40	31.40	45.60
		ZMP	63.60	68.60	68.60	54.40
	100	ZMPS_h	33.20	30.80	30.80	40.80
		ZMP	66.80	69.20	69.20	59.20
	200	ZMPS_h	40.00	39.40	39.40	44.80
		ZMP	60.00	60.60	60.60	55.20
	500	ZMPS_h	51.20	51.00	51.00	54.00
		ZMP	48.80	49.00	49.00	46.00
4	50	ZMPS_h	34.00	29.80	29.80	40.60
		ZMP	66.00	70.20	70.20	59.40
	100	ZMPS_h	38.00	36.60	36.60	43.60
		ZMP	62.00	63.40	63.40	56.40
	200	ZMPS_h	46.40	46.60	46.60	49.80
		ZMP	53.60	53.40	53.40	50.20
	500	ZMPS_h	56.20	57.20	57.20	60.40
		ZMP	43.80	42.80	42.80	39.60

Table 41: Estimates of correct selection probability when ZMPS_{u} is the true model (zero-inflation).

Scenario	n	Fitted	% DIC	% EAIC	% EBIC	% LMPL
1	50	ZMPS_{u}	74.80	74.80	74.80	81.40
		ZMP	25.20	25.20	25.20	18.60
	100	ZMPS_{u}	89.40	89.40	89.40	91.80
		ZMP	10.60	10.60	10.60	8.20
	200	ZMPS_{u}	98.00	98.00	98.00	98.00
		ZMP	2.00	2.00	2.00	2.00
	500	ZMPS_{u}	100.00	100.00	100.00	100.00
		ZMP	0.00	0.00	0.00	0.00
2	50	ZMPS_{u}	96.40	96.40	96.40	97.00
		ZMP	3.60	3.60	3.60	3.00
	100	ZMPS_{u}	99.80	99.80	99.80	99.80
		ZMP	0.20	0.20	0.20	0.20
	200	ZMPS_{u}	100.00	100.00	100.00	100.00
		ZMP	0.00	0.00	0.00	0.00
	500	ZMPS_{u}	100.00	100.00	100.00	100.00
		ZMP	0.00	0.00	0.00	0.00
3	50	ZMPS_{u}	64.20	64.00	64.00	75.80
		ZMP	35.80	36.00	36.00	24.20
	100	ZMPS_{u}	48.00	45.60	45.60	61.20
		ZMP	52.00	54.40	54.40	38.80
	200	ZMPS_{u}	33.20	30.80	30.80	43.40
		ZMP	66.80	69.20	69.20	56.60
	500	ZMPS_{u}	37.40	35.20	35.20	45.60
		ZMP	62.60	64.80	64.80	54.40
4	50	ZMPS_{u}	32.80	28.20	28.20	42.00
		ZMP	67.20	71.80	71.80	58.00
	100	ZMPS_{u}	33.40	32.20	32.20	41.00
		ZMP	66.60	67.80	67.80	59.00
	200	ZMPS_{u}	44.20	43.00	43.00	48.40
		ZMP	55.80	57.00	57.00	51.60
	500	ZMPS_{u}	53.60	53.60	53.60	55.80
		ZMP	46.40	46.40	46.40	44.20

Table 42: Estimates of correct selection probability when ZMPS_u is the true model (zero-deflation).

Scenario	n	Fitted	% DIC	% EAIC	% EBIC	% LMPL
1	50	ZMPS_u	56.00	56.00	56.00	62.40
		ZMP	44.00	44.00	44.00	37.60
	100	ZMPS_u	68.60	68.40	68.40	71.40
		ZMP	31.40	31.60	31.60	28.60
	200	ZMPS_u	77.80	77.80	77.80	79.80
		ZMP	22.20	22.20	22.20	20.20
	500	ZMPS_u	92.60	92.60	92.60	93.20
		ZMP	7.40	7.40	7.40	6.80
2	50	ZMPS_u	63.20	63.60	63.60	67.00
		ZMP	36.80	36.40	36.40	33.00
	100	ZMPS_u	72.60	72.80	72.80	75.60
		ZMP	27.40	27.20	27.20	24.40
	200	ZMPS_u	83.40	83.60	83.60	85.40
		ZMP	16.60	16.40	16.40	14.60
	500	ZMPS_u	96.80	96.80	96.80	96.80
		ZMP	3.20	3.20	3.20	3.20
3	50	ZMPS_u	30.20	25.80	25.80	38.40
		ZMP	69.80	74.20	74.20	61.60
	100	ZMPS_u	36.20	35.20	35.20	40.60
		ZMP	63.80	64.80	64.80	59.40
	200	ZMPS_u	43.80	44.00	44.00	49.40
		ZMP	56.20	56.00	56.00	50.60
	500	ZMPS_u	53.20	53.40	53.40	55.60
		ZMP	46.80	46.60	46.60	44.40
4	50	ZMPS_u	35.40	33.00	33.00	42.00
		ZMP	64.60	67.00	67.00	58.00
	100	ZMPS_u	44.20	43.80	43.80	50.80
		ZMP	55.80	56.20	56.20	49.20
	200	ZMPS_u	51.40	51.40	51.40	54.40
		ZMP	48.60	48.60	48.60	45.60
	500	ZMPS_u	61.20	61.60	61.60	64.40
		ZMP	38.80	38.40	38.40	35.60

1.3 Study 3

The following figures illustrate the results obtained in the third simulation study, described in Subsection 6.3 of the paper. In each figure, the horizontal dashed lines (red) were placed at 90% BRE. The vertical dashed lines (blue) were placed at $\theta = 0.18$ in Figure 1, at $\theta = 0.21$ in Figure 2 and at $\theta = 0.31$ in Figure 3.

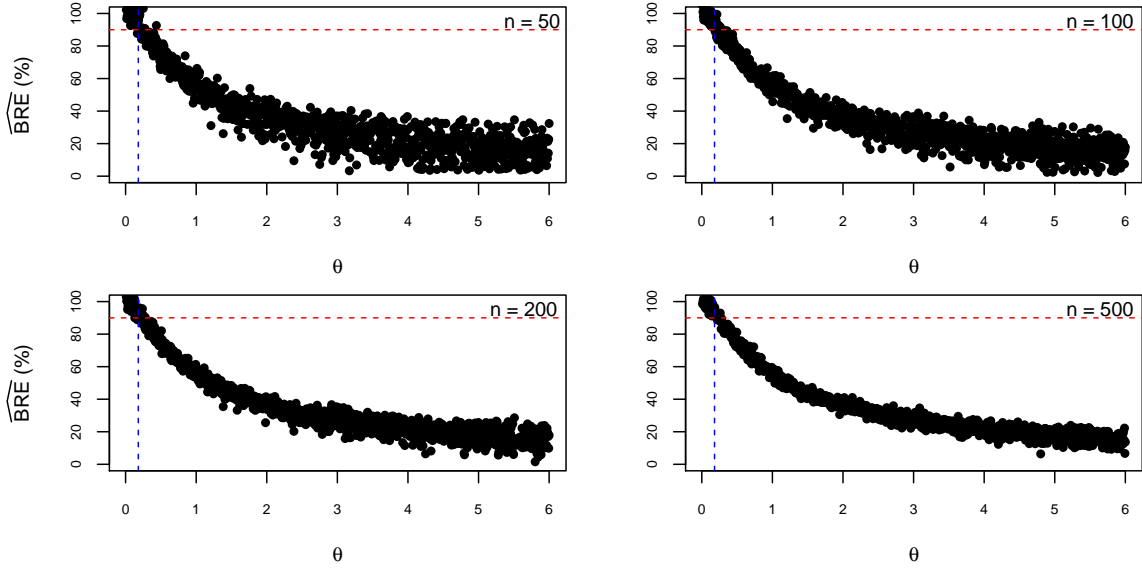


Figure 1: BRE of the ZMPL estimation when PL in the true model.

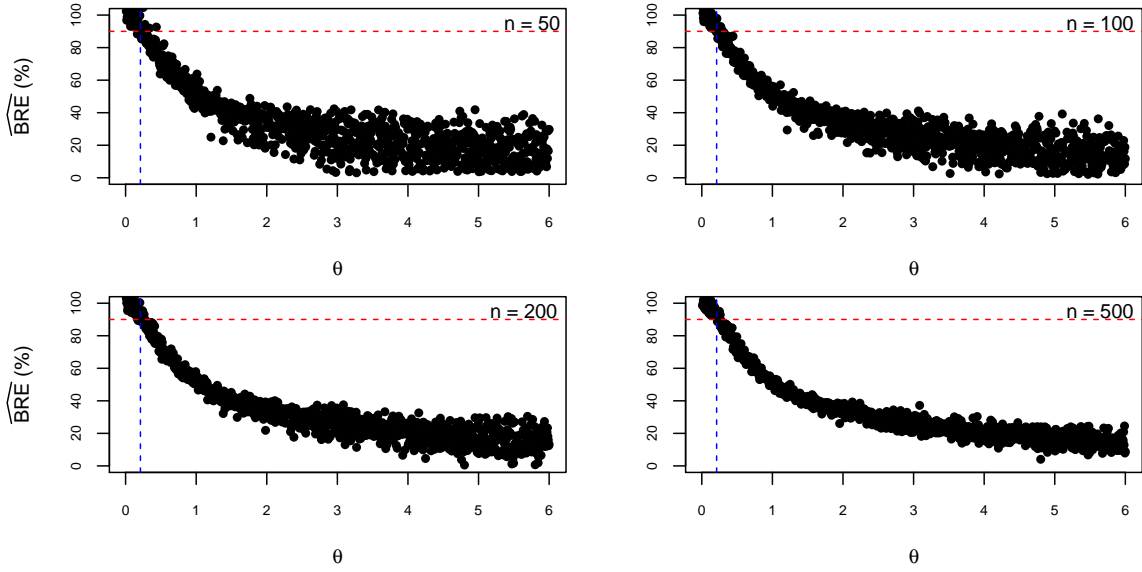


Figure 2: BRE of the ZMPS_h estimation when PS_h in the true model.

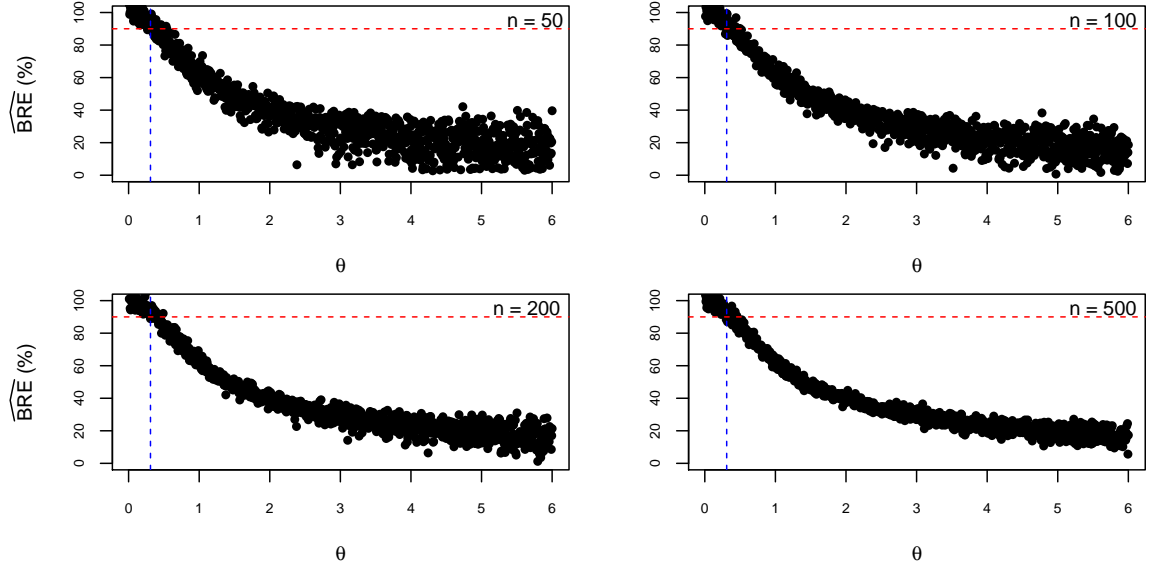


Figure 3: BRE of the ZMPS_u estimation when PS_u in the true model.

2 Datasets

The three real datasets used in the paper to illustrate the usefulness of the proposed class of models are provided in Table 43. A brief description of each dataset is presented in Section 7 of the manuscript.

Table 43: Real datasets used in the paper.

Dataset 1																			
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	2	2	2	2	2	2	2	2	2	2	2	3	3	3	3	3	3	3
4	4	4	4	5	5	5	6	6	6	6	6	6	7	8	17				
Dataset 2																			
0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2
2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
2	2	2	3	3	3	3	3	3	3	3	3	3	3	3	4	4	4	4	4
4	5	6	6	7	10														
Dataset 3																			
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	3	3
3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	4	4	4	5